

## Dynamics of Capture in the Restricted Three-Body Problem

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**Abstract.** We propose a new dynamical model for capture of irregular moons which identifies chaos as the essential feature responsible for initial temporary gravitational trapping within a planet’s Hill sphere. The key point is that incoming potential satellites get trapped in chaotic orbits close to “sticky” KAM tori in the neighbourhood of the planet, possibly for very long times, so that the chaotic layer largely dictates the final orbital properties of captured moons.

### 1. Introduction

The often puzzling properties of the irregular satellites of the giant planets—most of which have been discovered during the last six years (see Gladman et al. 2001; Hamilton 2003; Sheppard & Jewitt 2003; references therein and IAU Circular 8193)—provide a window into conditions in the early Solar System.

The general mechanism by which the irregular satellites were captured is thought to involve the following steps (Heppenheimer & Porco 1977; Pollack et al. 1979; Murison 1989; Peale 1999; Gladman et al. 2001); (i) temporary trapping close to the planet in a region roughly demarked by the Lagrange points  $L_1$  and  $L_2$ ; (ii) gradual energy loss through dissipation (e.g., gas drag or planetary growth) which translates temporary trapping into permanent capture; and (iii) possible fragmentation due to collisions at much later times. The hypothesis that the observed clustering among populations of irregular moons may be a result of fragmentation (Pollack et al. 1979; Gladman et al. 2001) contradicts, however, the fact that the orbits of known irregulars are clustered in inclination but not necessarily in eccentricity or other orbital elements (Nesvorný et al. 2003). Although there have been extensive studies of how the systems of irregular satellites have formed (see also Hénon 1970; Colombo & Franklin 1971; Huang & Innanen 1983; Saha & Tremaine 1993; Gor’kav’yi & Taidakova 1995; Marzari & Scholl 1998; Namouni 1999; Viera Neto & Winter 2001; Winter & Viera Neto 2001; Carruba et al. 2002; Carruba et al. 2003; Nesvorný et al. 2002; Nesvorný et al. 2003; Winter et al. 2003) a coherent *dynamical* picture of capture has not emerged; e.g., it has been widely held that the propensity for retrograde motion among Jupiter’s irregulars is simply due to the well known enhanced stability of retrograde orbits with large semimajor axes  $a$  (Nesvorný et al. 2003). Gladman et al. (2001) have called this, and the alternative “pull-down” (Heppenheimer

& Porco 1977) capture mechanism, into question based on the following observation: while the bulk of Jupiter’s irregular moons are retrograde and lie distant from the planet, Saturn’s cortege contains a more even mix of prograde and retrograde moons even though they have similarly large semimajor axes  $a$  when expressed in planetary radii. Here, we study capture in the circular restricted three-body problem (CRTBP) in two and three-dimensions (3D) taking the Sun–Jupiter–moon system as the specific example: the dynamical picture that emerges in the Hill limit (Murray & Dermot 1999; Simó & Stuchi 2000) is, however, rather similar for the other giant planets.

## 2. The Hamiltonian

In a coordinate system rotating with the mean motion of the primaries, but with origin transformed to the planet, the CRTBP Hamiltonian is given by

$$H = E = \frac{1}{2}\mathbf{p}^2 - (x p_y - y p_x) - \frac{\mu}{\sqrt{x^2 + y^2 + z^2}} - \frac{1 - \mu}{\sqrt{(1 + x)^2 + y^2 + z^2}} - (1 - \mu)x + \alpha, \quad (1)$$

where  $a$  is scaled to 1,  $\mu = m_1/(m_1 + m_2)$ ;  $m_1$  and  $m_2$  are the masses of the primaries and  $\alpha$  is a collection of inessential constants retained for consistency in relating the energy  $E$  to the Jacobi constant  $C_J = -2E$ ;  $\mathbf{r} = (x, y, z)$  and  $\mathbf{p} = (p_x, p_y, p_z)$  are the coordinates and momenta of the potential satellite. This Hamiltonian is obtained from the standard CRTBP Hamiltonian (Murray & Dermot 1999) by the canonical transformation  $x \rightarrow x' + (1 - \mu)$ ,  $p_y \rightarrow p'_y + (1 - \mu)$  (and dropping the primes). Angular momentum,  $\mathbf{h} = (h_x, h_y, h_z)$ , where  $h_z = x p_y - y p_x$ , is now defined with respect to the planet as is natural for a study of capture.

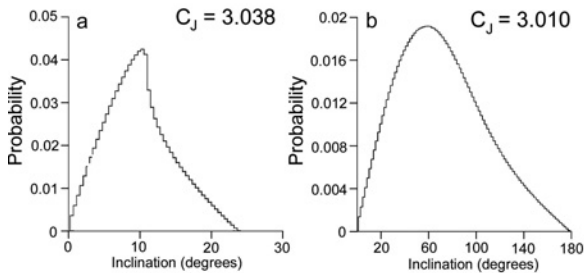


Figure 1. Histograms of inclination distribution for  $10^8$  test particles originating at the Hill sphere for indicated values of energy (Jacobi constant).

### 3. Simulations

Figure 1 shows computed orbital inclination distributions [ $I = \arccos(h_z/h)$ ] for a flux of  $10^8$  test particles as they pass through the Hill sphere [radius  $R_H = a(\mu/3)^{1/3}$ , Murray & Dermot 1999] at two energies. The key observation is that at low energy (Figure 1a) only *prograde* orbits can enter (or exit) the capture zone between Lagrange points  $L_1$  and  $L_2$  whereas at higher energies (Figure 1b) the distribution shifts to include both senses of  $h_z$ . This is because not all parts of the Hill sphere are energetically accessible at low energies. Figure 1 thus suggests that the statistics of capture might be expected to depend on initial  $C_J$  and  $\mathbf{h}$ .

To investigate this we consider first the structure of phase space in the planar limit ( $z = p_z = 0$ ). Figure 2 displays a series of Poincaré surfaces of section (SOS) for randomly chosen initial conditions inside the Hill radius  $R_H$ . The hypersurface is the  $x$ - $y$  plane with units rescaled to  $R_H = 1$  and points colored according to the sign of angular momentum (grey, retrograde  $h_z < 0$ ; black, prograde  $h_z > 0$ ) as they intersect the surface with  $p_x = 0$  and  $dy/dt > 0$ .

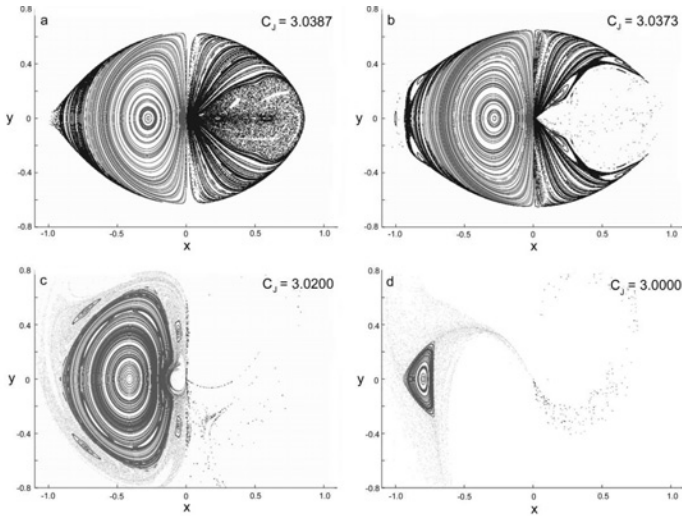


Figure 2. Poincaré surfaces of section for Sun–Jupiter–satellite 2D CRTBP with increasing energy.

With increasing energy a coherent dynamical picture emerges; in Figure 2a the prograde orbits exhibit regions of strongly chaotic motion whereas *all* the retrograde orbits are regular (quasiperiodic). This forces incoming prograde orbits to remain prograde because they cannot penetrate the KAM regions in Figure 2a. After the gateway at  $L_2$  has opened in Figure 2b the chaotic “sea” of prograde orbits visible in Figure 2a rapidly disappears except for a thin residual front of chaos which sticks to the KAM tori, separating them from the growing basin of direct scattering. As energy increases further this front moves from

prograde to retrograde motion while the tori steadily erode. KAM tori are “sticky” and chaotic orbits near them can appear locally near-integrable, i.e., they are trapped in almost regular orbits for very long times. Note especially that the KAM tori in Figure 2c and 2d exist at energies well *above*  $L_1$  and  $L_2$ . Permanent capture happens if dissipation is sufficient to switch long lived chaotic orbits into KAM regions which means that chaotic orbits can be permanently captured, even above the saddle points by relatively weak dissipation.

The SOS also reveal that large distance from the planet need not imply retrograde motion: e.g., the orbits visible inside two KAM islands centered at  $x > 0, y = 0$  in Figure 2a are large, almost circular, periodic *prograde* orbits but whose centers are displaced from the origin. At higher energies capture (through dissipation) is into retrograde KAM surfaces nested around the circular, retrograde orbit ( $x < 0, y = 0$ ) (Hénon 1970; Winter & Viera Neto 2001), and which remains almost perfectly centered on the planet.

In 3D, initial conditions in  $\mathbf{r}$  were chosen uniformly and randomly on the Hill sphere with random velocities. The Jacobi constant was also chosen randomly and uniformly  $C_J \in (2.995, C_J^{L_1})$ . Trajectories were integrated until they exited the Hill sphere, came within 2 planetary radii of the origin (Carruba et al. 2002) or survived for a predetermined cutoff time  $t_{\text{cut}}$ . Computational limitations restricted us to 80 million test particles, of which 23000 survived for  $t_{\text{cut}} = 20000$  years. Figure 3a shows a clear trend from prograde to retrograde capture with increasing energy. Three main islands stand out in the archipelago visible in Figure 3a; the prograde (low inclination) island shrinks noticeably with increasing  $t_{\text{cut}}$  reflecting the lower probability of prograde capture.

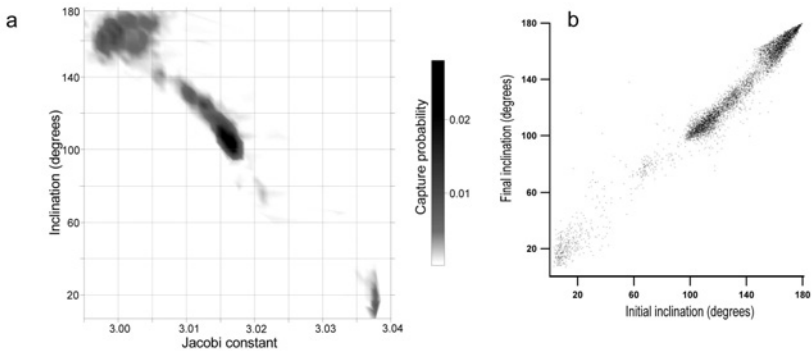


Figure 3. (a) The normalized capture probability distribution from Monte Carlo simulations (Jupiter), and (b) correlation between initial and final inclination of the test particles trapped in the capture zone.

The most noticeable feature is the large island at  $I \approx 100^\circ$  whose stability is related to the Kozai resonance centered at  $I = 90^\circ$  (Kozai 1962; Innanen et al. 1997; Carruba et al. 2002; Carruba et al. 2003; Nesvorný 2003). Unlike in 2D direct injection into KAM regions is possible but in near-integrable regions occurs exponentially slowly which seems to be why incoming particles are excluded from the center of the resonance itself. Likely these particles are trapped

in a chaotic separatrix layer of the Kozai resonance. Both the “Kozai island” and the smaller, very high energy island of retrograde motion in Figure 3a are stable for extremely long times. Figure 3b demonstrates a very strong correlation between final and initial inclination (i.e., “inclination memory”, Astakhov et al. 2003) for long lived orbits which is consistent with the picture in 2D.

We have confirmed numerically that these distributions are similar for the other giant planets; are robust in the presence of gas drag of different forms. The specific details of capture depend sensitively on how this dynamical mechanism intersects local environment around the planet. One of the factors is the part of the Hill sphere occupied by the massive regular moons. This observation is important, because prograde orbits penetrate much deeper towards the planet than do most of the retrograde orbits (see Figures 2b, 2d and also Simó & Stuchi 2000). Therefore, to be permanently captured, prograde satellites must survive close encounters or collisions with “influential” regular moons. In our Monte Carlo simulations with dissipation (Astakhov et al. 2003) we eliminated test particles that crossed the orbits of Titan at Saturn and Callisto at Jupiter. These simulations indicate that Saturn/Titan in tandem have a clear tendency to capture a higher ratio of prograde to retrograde moons as compared to Jupiter/Callisto. This is because Callisto’s orbit represents a larger fraction of the Hill sphere than does Titan’s orbit. Thus the relative scarcity of jovian prograde irregulars may be due to potential prograde satellites having been swept away more efficiently by Jupiter’s Galilean moons (Astakhov et al. 2003).

**Acknowledgments.** This work was funded by grants from the US National Science Foundation and Petroleum Research Fund to D.F.

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